Phase Equilibria in the System Fe-Fe₂O₃-Lu₂O₃ at 1200°C

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Phase equilibria in the system $Fe_{-}Fe_{2}O_{3}$ -Lu₂O₃ have been established at 1200°C. The following phases were stable: lutetium sesquioxide, hematite, magnetite, wustite, metallic iron, lutetium-iron-perovskite, lutetium-iron-garnet, LuFe₂O₄, and Lu₂Fe₃O₇. The standard free energy of formation from the metallic iron, Lu₂O₃, and oxygen at 1200°C of LuFe₂O₄, Lu₂Fe₃O₇, LuFeO₃, and Lu₃Fe₅O₁₂ has been determined to be $-100.3 \pm 0.2, -158.2 \pm 0.2, -57.5 \pm 0.3$, and -279.1 ± 0.9 kcal/mole, respectively.

Introduction

In previous studies, Kimizuka and Katsura (1), Sugihara et al. (2), and Kimizuka and Katsura (3, 4) have established the phase equilibria in the $Fe-Fe_2O_3-Ln_2O_3$ systems (Ln = La, Y, Eu, and Yb) at 1200°C. They have also determined the standard free energy of formation from metallic iron, lanthanoid sesquioxide, and oxygen of ternary double oxides found in these systems. Kimizuka and Katsura (4) have proposed that the Fe- Fe_2O_3 -Ln₂O₃ systems (Ln = La, Nd, Sm, Eu, Gd, Tb, Dy, Ho, Er, Tm, Yb, Lu, and Y) could be classified into four groups with respect to the assemblage of ternary compounds which are in stable coexistence at 1200°C. According to them, the Fe-Fe₂O₃- Lu_2O_3 system may belong to the *D*-type having four stable ternary compounds, that is, $LuFeO_3$, $Lu_3Fe_5O_{12}$, $LuFe_2O_4$, and $Lu_2Fe_3O_7$.

The objectives of the present study are, firstly, to test the previous assumption about the classification of the Fe–Fe₂O₃–Lu₂O₃ system at 1200°C after establishing the phase equilibria in this system, and secondly, to determine the standard free energy of formation from the metallic iron, Lu₂O₃, and oxygen.

Experimental

Guaranteed reagent grade Fe_2O_3 and Lu_2O_3 (99.99%) powders were employed for all starting materials. Desired ratios of Lu_2O_3/Fe_2O_3 were obtained by mixing thoroughly in an agate mortar under ethyl alcohol. Mixtures thus obtained were treated by the same procedures as those described previously (4).

Apparatus and procedures for controlling the oxygen partial pressure, keeping constant temperature, method of thermogravimetry, the criterion for equilibrium establishment, the method of identification of solid phases after quenching, and the method of wet chemical analysis are the same as those described in previous papers (I-4). Thus, the range of experimental errors in determining the standard free energy is in the same order of magnitude as those of previous results.

Results and Discussion

1. Phase Equilibria

Figure 1 shows the phase equilibria in the $Fe-Fe_2O_3-Lu_2O_3$ system at 1200°C. The following phases were stable under the experimental conditions used: lutetium sesquioxide



FtG. 1. The phase diagram of the Fe–Fe₂O₃–Lu₂O₃ system at 1200°C. Values in area of three solid phase equilibria mean –log P_{O2} (atm). The composition corresponding to each letter is given in Table III. Open circles show the experimental results. A boundary line M_1 – P_2 by which the area $G_1 + M_1 + P_2$ is separated from the area $B_4 + M_1 + P_2$ is, strictly drawn, composed of two boundary lines: One is M_1 – P_2 , and the other M_1 – P_2' . The composition of P_2' is very slightly shifted to the direction of stoichiometric perovskite P. The small area for M_1 + (solid solution from P_2 to P_2') was recognized experimentally.

(Lu₂O₃), hematite (Fe₂O₃), magnetite (Fe₃O₄), lutetium-iron-perovskite (LuFeO₃), lutetiumiron-garnet (Lu₃Fe₅O₁₂), LuFe₂O₄, Lu₂Fe₃O₇, wustite (FeO), and γ -Fe.

Lutetium sesquioxide was stable at oxygen partial pressures from 1 to 10^{-16} atm and showed no significant nonstoichiometry in its composition. The volatility of Lu₂O₃ at 1200°C over the experimental range of oxygen partial pressures was almost the same order of magnitude as that of Yb₂O₃ studied by Kimizuka and Katsura (4) and was determined to be approximately $2 \text{ mg}/3 \text{ g of } \text{Lu}_2\text{O}_3$ during heating for 3 days.

The deviation from the stoichiometric composition of lutetium-iron-perovskite ranged up to LuFeO_{2.958} on the basis of the weight change. The lattice constants of the LuFeO₃ phase are given in Table I on the basis of an orthorhombic crystal system. As seen in Table I, no significant effects on the lattice constants due to the nonstoichiometry were observed. This is the same trend observed in EuFeO₃ (2), YFeO₃ (3), and YbFeO₃ (4).

ΤA	BL	Æ	Ł

Composition	0	h	c (Å)	V (Å ³)
Composition	**			-
LuFeO ₃ "				
LuFeO _{3,000}	5.2123 ± 0.0003	5.5531 ± 0.0005	7.5622 ± 0.0006	218.88 ± 0.03
LuFeP _{2,960}	5.2130 ± 0.0004	5.5555 ± 0.0006	7.5609 ± 0.0007	218.97 ± 0.03
$Lu_3Fe_5O_{12}^b$				
$Lu_{3}Fe_{5}O_{12,00}$	12.2783 ± 0.0003			1851.1 ± 0.1
Lu ₃ Fe ₅ O _{11.96}	12.2786 ± 0.0002			1851.2 ± 0.1
$Lu_2Fe_3O_7$				
$Lu_2Fe_3O_{7.000}$	3.4537 ± 0.0001		28.427 ± 0.003	293.65 ± 0.04
Lu ₂ Fe ₃ O _{6,997}	3.4493 ± 0.0001		28.434 ± 0.003	292.98 ± 0.04
Lu ₂ Fe ₃ O _{6.984}	3.4527 ± 0.0001		28.420 ± 0.002	293.40 ± 0.03
$Lu_2Fe_3O_{6,927}$	3.4539 ± 0.0001		28.434 ± 0.002	293.76 ± 0.04
$Lu_2Fe_3O_{6,896}$	3.4527 ± 0.0004		28.449 ± 0.008	294.4 ± 0.1
LuFe ₂ O ₄				
$LuFe_2O_{4,015}$	3.4261 ± 0.0002		25.294 ± 0.004	257.13 ± 0.06
$LuFe_2O_{4,007}$	3.4305 ± 0.0001		25.278 ± 0.002	257.63 ± 0.03
$LuFe_{2}O_{3,998}$	3.4353 ± 0.0002		25.250 ± 0.003	258.06 ± 0.05
LuFe ₂ O _{3.982}	3.4373 ± 0.0001		25.260 ± 0.003	258.47 ± 0.03
LuFe ₂ O _{3.951}	3.4390 ± 0.0001		25.267 ± 0.002	258.80 ± 0.03
LuFe ₂ O _{3.935}	3.4417 ± 0.0002		25.266 ± 0.004	259.19 ± 0.05

Lattice Constants of LuFeO3, Lu_3Fe_5O12, Lu_2Fe_3O7 and LuFe_2O4

" $a = 5.213 \pm 0.003$, $b = 5.547 \pm 0.003$, $c = 7.565 \pm 0.003$ (12).

 $^{h}a = 12.283 \pm 0.003$ (13).

As seen in Fig. 1, lutetium-iron-garnet has a significant range of nonstoichiometry up to $Lu_3Fe_5O_{11.96}$. However, no effect was observed on the lattice constant, and the data are given in Table 1. In previous studies, it was hard to find the existence of nonstoichiometric lanthanoid-iron-garnet (Eu, Yb, and Y). Perhaps, the $Lu_3Fe_5O_{12}$ phase may be the only case having any significant nonstoichiometry.

The LuFe₂O₄ and Lu₂Fe₃O₇ solid solutions have wide compositional ranges at 1200°C, as shown in Fig. 1. The compositional range of the LuFe₂O₄ solid solution was from LuFe₂-O_{3.935} to LuFe₂O_{4.018} and that of the Lu₂Fe₃O₇ was from Lu₂Fe₃O_{6.879} to Lu₂Fe₃-O_{7.000}. Both phases are black and neither was attracted by a hand magnet at room temperature. The *hkl* indices of the X-ray powder patterns of LuFe₂O₄ were obtained on the basis of the results by Kato et al. (5), and that of Lu₂Fe₃O₇ by Kimizuka et al. (6) and Kato et al. (7). The change of lattice constants of both the solid solutions with composition is given in Table I based on a hexagonal crystal system. It is interesting that the *c* value in the LuFe₂O₄ solid solution changes with composition so that it has a minimum at the stoichiometric composition.

Kimizuka and Katsura (4) have classified the Fe-Fe₂O₃- Ln_2O_3 systems (Ln = La, Nd, Sm, Eu, Gd, Tb, Dy, Ho, Er, Tm, Yb, Lu, and Y) into four groups with respect to the assemblage of ternary compounds as: Type A has only one stable compound $LnFeO_3$ (Ln = La and Nd); type B, $LnFeO_3$ and $Ln_3Fe_5O_{12}$ (Ln = Sm, Eu, Gd, Tb, and Dy); type C, $LnFeO_3$, $Ln_3Fe_5O_{12}$, and $LnFe_2O_4$ (Ln = Ho, Er, Tm, and Y); and type D, $LnFeO_3$, $Ln_3Fe_5O_{12}$, $LnFe_2O_4$, and $Ln_2Fe_3O_7$ (Ln = Yb and Lu). Indeed, the Fe-Fe₂O₃- Lu_2O_3 belongs to the D type, as illustrated in Fig. 1, but it is noteworthy that there is some basic difference in phase diagrams between these two systems classified into the D type. In the case of the $Fe-Fe_2O_3-Lu_2O_3$ system, the extremely oxidized Lu₂Fe₃O₇ phase is in equilibrium with both the LuFeO₃ and Fe₃O₄ phases at an oxygen partial pressure of $10^{-7.53}$ atm. On the contrary, in the Fe-Fe₂O₃-Yb₂O₃ system, the extremely oxidized Yb₂Fe₃O₇ phase can never be in equilibrium with the Fe₃O₄ phase, but is in equilibrium with both the YbFeO₃ and the extremely oxidized YbFe₂O₄ phases. Thus, the D-type is rather complicated in comparison with the other three types.

2. Calculations of the Standard Free Energy of Formation of Ternary Compounds from Metallic iron, Lu₂O₃, and Oxygen

On the basis of the present phase equilibria, we will determine the standard free energy of formation of the compounds $LuFe_2O_4$, $Lu_2Fe_3O_7$, $LuFeO_3$, and $Lu_3Fe_5O_{12}$, referred to the following equations:

$$2Fe + 1/2Lu_2O_3 + 5/4O_2 = LuFe_2O_4$$
(1)

$$3Fe + Lu_2O_3 + 2O_2 = Lu_2Fe_3O_7$$
 (2)

$$Fe + 1/2Lu_2O_3 + 3/4O_2 = LuFeO_3$$
 (3)

$$5Fe + 3/2Lu_2O_3 + 15/4O_2 = Lu_3Fe_5O_{12}$$
. (4)

Now, Fig. 1 makes it possible to calculate the standard free energy of reactions (5) to (12) tabulated in Table II. Once we could determine the standard free energy of these reactions, (5)-(12), then the standard free energy of formation referred to each equation, (1)-(4), is readily calculated as

$$\Delta G^{\circ}(1) = 2\Delta G^{\circ}(5) + \Delta G^{\circ}(6) \tag{1'}$$

$$\Delta G^{\circ}(2) = 3/2 \Delta G^{\circ}(1) + \Delta G^{\circ}(7)$$
 (2')

$$= \Delta G^{\circ}(10) - 1/3 \Delta G^{\circ}(9) + 2 \Delta G^{\circ}(1) (2'')$$

$$\Delta G^{\circ}(3) = 1/3 \Delta G^{\circ}(2) + \Delta G^{\circ}(8)$$
 (3')

$$= \frac{1}{2}\Delta G^{\circ}(11) - \frac{1}{6}\Delta G^{\circ}(9) + \frac{1}{2}\Delta G^{\circ}(2)$$
 (3")

$$\Delta G^{\circ}(4) = 2\Delta G^{\circ}(5) + 3\Delta G^{\circ}(3) + 2/3\Delta G^{\circ}(9) + \Delta G^{\circ}(12).$$
(4')

To determine the standard free energy of the reactions referred to each equation, (5)-(12), it is necessary to calculate the activity of each component, FeO, Fe₃O₄, LuFe₂O₄, Lu₂Fe₃O₇, and LuFeO₃. Here, the reference activity of each component in each solid solution was set equal to one where the composition of the solid solution reached the most reduced composition, that is, $a_{FeO} = 1$ where its composition is FeO_{1.049} (W₁ in Fig. 1), $a_{Fe_3O_4} = 1$ at Fe₃O_{4.000} (M₁ in Fig. 1), $a_{LuFe_2O_4} = 1$ at LuFe₂O_{3.935} (A₁ in Fig. 1), $a_{Lu_5Fe_3O_7} = 1$ at Lu₂Fe₃O_{6.879} (B₁ in Fig. 1),

SUMMARY OF THE STANDARD FREE ENERGY VALUES

	Reactions	$-\Delta G^{\circ}$ (kcal)	
(1)	2 Fe + $1/2$ Lu ₂ O ₃ + $5/4$ O ₂ = LuFe ₂ O ₄	100.3 ± 0.2	
(2)	$3 \text{ Fe} + Lu_2O_3 + 2 O_2 = Lu_2\text{Fe}_3O_7$	158.2 ± 0.2 (mean value) ^{<i>a</i>}	
(3)	$3 \text{ Fe} + 1/2 \text{ Lu}_2\text{O}_3 + 3/4 \text{ O}_2 = \text{LuFeO}_3$	57.5 ± 0.3 (mean value) [*]	
(4)	$5 \text{ Fe} + 3/2 \text{ Lu}_2\text{O}_3 + 15/4 \text{ O}_2 = \text{Lu}_3\text{Fe}_5\text{O}_{12}$	279.1 ± 0.9	
(5)	$Fe + 1/2 O_2 = FeO$	40.2 ± 0.2	
(6)	$2 \text{ FeO} + \frac{1}{2} \text{ Lu}_2 \text{O}_3 + \frac{1}{4} \text{ O}_2 = \text{LuFe}_2 \text{O}_4$	19.7 ± 0.1	
(7)	$3/2 \text{ LuFe}_2\text{O}_4 + 1/4 \text{ Lu}_2\text{O}_3 + 1/8 \text{ O}_2 = \text{Lu}_2\text{Fe}_3\text{O}_7$	7.79 ± 0.1	
(8)	$1/3 Lu_2Fe_3O_7 + 1/6 Lu_2O_3 + 1/12 O_2 = LuFeO_3$	4.85 ± 0.1	
(9)	$3 \text{ FeO} + 1/2 \text{ O}_2 = \text{Fe}_3 \text{O}_4$	33.8 ± 0.3	
(10)	$2 \text{LuFe}_{2}\text{O}_{4} + \frac{1}{6}\text{O}_{2} = \text{Lu}_{2}\text{Fe}_{3}\text{O}_{7} + \frac{1}{3}\text{Fe}_{3}\text{O}_{4}$	8.84 ± 0.1	
(11)	$1/2 Lu_2Fe_3O_7 + 1/12 O_2 = LuFeO_3 + 1/6 Fe_3O_4$	4.18 ± 0.1	
(12)	$3 \text{ LuFeO}_3 + 2/3 \text{ Fe}_3\text{O}_4 + 1/6 \text{ O}_2 = \text{Lu}_3\text{Fe}_5\text{O}_{12}$	3.55 ± 0.2	
. ,			

^a 158.3 from (2'), 158.0 from (2'').

^b 57.6 from (3'), 57.4 from (3'').





FIG. 2a. The relationship between $-\frac{1}{2} \log P_{O_2}$ and $N_o/N_{LuFc_2O_4}$.

FIG. 2b. The relationship between $-\frac{1}{2} \log P_{O_2^{\perp}}$ and $N_o/N_{Lu_2Fc_3O_7}$.

Component (i)	Composition	Symbol	$-\log P_{O_2}$	$\log a_i$
FeO	FeO _{1.049}	\mathbf{W}_1	11.94	0
	FeO _{1.064}	W ₂	11.58	-0.0107
	FeO _{1.165}	W ₃	9.14	-0.1435
LuFe ₂ O ₄	LuFe ₂ O _{3,935}	A_1	11.58	0
2 4	$LuFe_2O_{3,982}$	A ₂	9.72	0.0397
	LuFe ₂ O _{4,000}	A ₃	9.14	0.0437
	$LuFe_2O_{4,018}$	A4	8.11	0.0414
$Lu_2Fe_3O_7$	Lu ₂ Fe ₃ O _{6.879}	B_1	9.72	0
	Lu ₂ Fe ₃ O _{6,944}	B ₂	8.81	0.0436
	Lu ₂ Fe ₃ O _{6.984}	B ₃	8.11	0.0540
	$Lu_{2}Fe_{3}O_{7.000}$	B_4	7.53	0.0527
LuFeO3	LuFeO _{2.958}	\mathbf{P}_1	8.81	0
	LuFeO2.985	P ₂	7.53	0.0184
	LuFeO _{2,999}	P_3	3.75	0.0649
	LuFeO _{3.000}	P_4	0.00	0.0650
$Lu_3Fe_5O_{12}$	Lu ₃ Fe ₅ O _{11,96}	G_1	3.75	0
	Lu3Fe5012 000	G,	0.00	

TABLE III Composition and Activity in Five Solid Solutions

and $a_{LuFeO_3} = 1$ at LuFeO_{2.958} (P₁ in Fig. 1). Desired activities to determine the standard free energy were calculated by applying the Gibbs-Duhem equation as described in previous papers (3, 4). The relationship between the equilibrium oxygen partial pressure and the composition in each solid solution, which is necessary to calculate the activity, was substantially linear in all series of solid solutions as illustrated, for example, in Figs. 2a and 2b. The calculated activity data are summarized in Table III, and the standard free energy data thus obtained are given in Table II. 3. The Relationship between the Standard Free Energy of Formation of Lanthanoid–Iron– Perovskite and Lanthanoid Sesquioxide, Metallic Iron, and Oxygen and the Tolerance Factor

In previous papers, Kimizuka and Katsura (4) and Katsura et al. (8) pointed out that the standard free energy of lanthanoid-ironperovskite (La, Nd, Sm, Eu, Gd, Tb, Dy, and Yb) changes in proportional to the tolerance factor defined by Goldschmidt (9) [tolerance factor $t = (r_a + r_o)/\sqrt{2}(r_b + r_o)$, where r_a , r_b , and r_{o} mean ionic radii of lanthanoid, ferric, and oxygen ions in the perovskite structure, respectively. We took the values of ionic radii reported by Espinosa (13) for lanthanoid ions with 12 coordination number in the perovskite structure, and the values of 0.645 and 1.40 Å for Fe^{3+} and O^{2-} ions, respectively, given by Shannon and Prewitt (14)]. This proportionality is also applicable to $LuFeO_3$, $HoFeO_3$ (10), and $ErFeO_3$ (11) with an exception of GdFeO₃, as seen in Fig. 3. More detailed



FIG. 3. The relationship between ΔG° and the tolerance factor *t*.

discussion on the relationship between the thermochemical properties of lanthanoidiron-perovskite and their structural stability will be published in the near future after completing the study of the whole series of lanthanoid-iron-perovskite.

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